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INVENTORY MODELS FOR AMELIORATING AND DETERIORATING ITEMS WITH TIME DEPENDENT DEMAND AND IHC

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Abstract
In this paper we have developed two inventory models for ameliorating and deteriorating items.
Model – I   : Inventory Model with two time intervals
Model – II : Inventory Model with three time intervals
In both these models we have considered time to ameliorate to follow Weibull distribution, deterioration rate to be constant and shortages are not allowed to occur. We have also suggested an algorithm to minimize total average cost for both these models. ***Model – I*** *is derived as a particular case of* ***Model – II*** *and it is illustrated by a numerical example with its sensitivity analysis.*

Keywords – Amelioration, Deterioration, Weibull Distribution.

1. Introduction

Currently many researches are being carried out in the field of inventory management. New inventory models are developed by changing deterioration, shortages and demand rates to obtain EOQ or EPQ. Ankit Bhojak and U. B. Gothi [1] and [2], Kirtan Parmar and U. B. Gothi [9], Kirtan Parmar, Indu Aggarwal and U. B. Gothi [10], Devyani Chatterji and U. B. Gothi [4] and [15] have developed various EOQ and EPQ models. Here a new dimension is added to such inventory models by introducing amelioration. In this research it has been studied that how changes occur on inventory model due to ameliorating and deteriorating items.

When items like fruits, green vegetables, flowers, some dairy products are either kept in farms, in supermarkets or in cold storages, their stock increases due to growth whereas at the same time the stock can also decrease due to various reasons like spoilage, decaying and so on. Such wastage or spoilage can be reduced by providing appropriate treatment in farm or by maintaining proper temperature in supermarket and at storage places. In such cases the combined effect of deterioration and amelioration are observed. Hwang [5], [6] and [7] has developed different inventory models for ameliorating items. An inventory system of ameliorating items for price dependent demand rate was given by Mondal, Bhunia and Maiti [3]. Moon, Giri and Ko [8] have presented EOQ models for ameliorating/deteriorating items under inflation and time discounting. Tadj, Sarhan and Gohary [11] obtained an optimal control of an inventory system with ameliorating and deteriorating items. Law and Wee [12] have derived an integrated production inventory model for ameliorating and deteriorating items considering time discounting. Inventory model with Weibull amelioration under the influence of inflation and time-value of money was derived by Mishra, Misra, Mallick and Barik [14]. Mishra, Raju, U.K. Misra and G. Misra [13] have published their work on optimal control of an inventory system with variable demand and ameliorating / deteriorating items.

We have derived two inventory models by considering amelioration of items as a combination of two parameter and three parameter Weibull distribution in the different time intervals. Deterioration rate is constant throughout cycle period.

Model – I: Inventory Model With Two Time Intervals
In this model we have considered demand as a linear function of time. This model is suitable for the products which are newly launched in the market, the demand of that product increases slowly. In such case demand can be considered as a linear function of time. But as the time passes when that product gains popularity in market, it is not appropriate to consider demand as a linear function of time as earlier. Therefore to cope up with such situation a new model (Model – II) is developed in which demand is considered as a quadratic function of time instead of linear one.

Model – II : Inventory Model With Three Time Intervals
In the third time interval of Model – II, demand rate is considered as a quadratic function depending on time.

Thereafter for both the developed models we have minimized the total average cost.
Model – I is illustrated by numerical example with its sensitivity analysis.

2. Assumptions for Both the Models

The models are developed under the following assumptions:

1. The inventory system involves only one item and one stocking point.
2. Replenishment rate is infinite.
3. Lead-time is zero.
4. The deterioration and amelioration occur when the item is effectively in stock.
5. The deteriorated items are not replaced during the given cycle.
6. Infinite time horizon period is considered.
7. Shortages are not allowed to occur.
8. Holding cost $C_h = h + r\,t$ $(h, r > 0)$ is a linear function of time.
9. $\theta(t) = \theta$ is a constant deterioration rate.
10. In Model – I demand rate $R(t) = a + b\,t$ is a linear function of time throughout the time interval [0, T] whereas in Model – II,

$$R(t) = \begin{cases} a + b\,t & 0 \le t \le t_1 \\ a + b\,t + c\,t^2 & t_1 \le t \le T \end{cases} \qquad (a, b, c > 0).$$

11. Amelioration cost, deterioration cost, production cost and ordering cost are known and constants.
12. For both the models, the amelioration rate

$$A(t) = \begin{cases} \sim \quad t^{\;-1} & 0 \le t \le « \\ \sim \quad (t - «)^{\;-1} & « \le t \le T \end{cases}$$

where α is a scale parameter ($0 < \alpha << 1$) and β is a shape parameter ($\beta > 0$)

13. Total inventory cost is a continuous real function which is convex to the origin.

3. Notations for both the Models

The following notations are used to develop the mathematical model:

1. Q (t) : Inventory level of the product at time t (≥ 0).
2. R (t) : Demand rate varying over time.
3. A (t) : Amelioration rate at any time t.
4. θ (t) : Deterioration rate.
5. A : Ordering cost per order during the cycle period.
6. C_h : Inventory holding cost per unit per unit time.
7. C_a : Amelioration cost per unit
8. C_d : Deterioration cost per unit.
9. C_p : Production cost per unit. ($C_p > C_a$)
10. S : Initial inventory level at time t = 0.
11. S_1 : Inventory level at time $t = \mu$.
12. S_2 : Inventory level at time $t = t_1$.
13. T : Duration of a cycle.
14. TC : Total cost per unit time.

4. Mathematical Formulation And Solution

Model – I : Inventory Model With Two Time Intervals

Cycle starts with inventory level of S units. The total time is divided into two time intervals. In the first time interval $[0, \mu]$ the inventory level decreases due to the combined effect of two parameter Weibull amelioration, deterioration and demand and it reaches to S_1 at $t = \mu$. In the second time interval $[\mu, T]$ the inventory level decreases and reaches to zero at time t = T due to the combined effect of three parameter Weibull amelioration, deterioration and demand. Deterioration rate is constant throughout time interval [0, T]. Demand rate is a linear function of time. The above mentioned inventory system is presented graphically in Fig. 1.

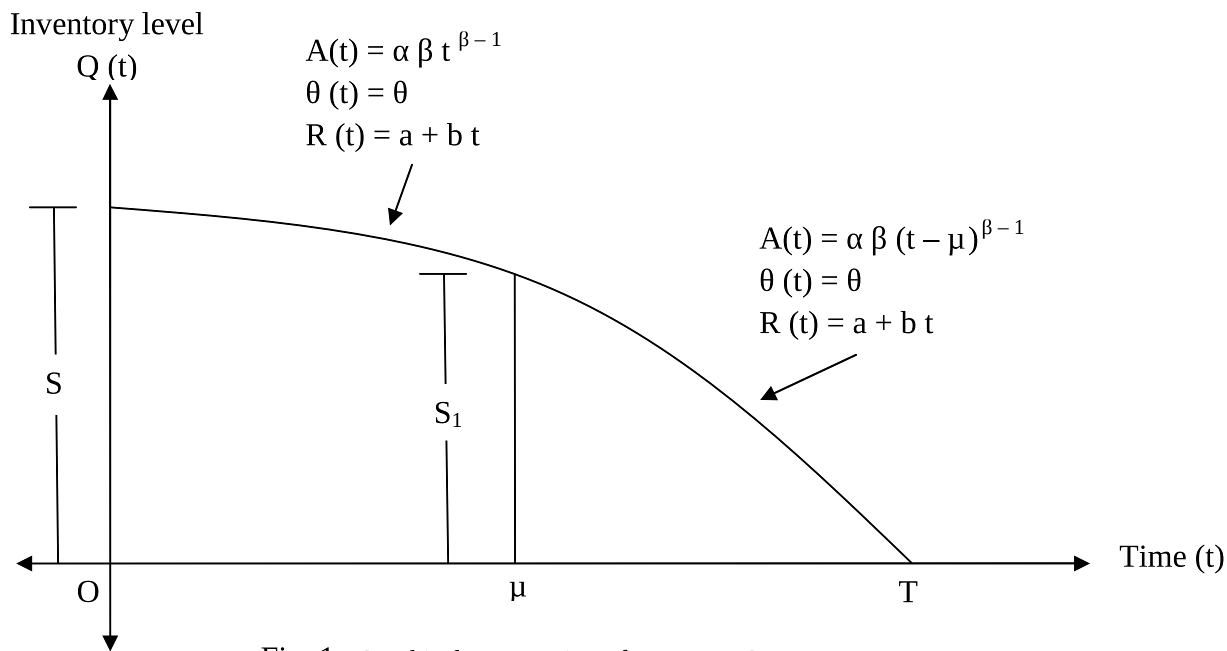

Fig. 1. Graphical presentation of Inventory System

Differential Equations pertaining to the situations as explained above are given by

$$\frac{d\,Q\,(t)}{d\,t} = \sim \quad t^{\;-1}\,Q\,(t) - \text{¯}\,Q\,(t) - (a+b\,t) \qquad 0 \le t \le \text{«} \qquad \dots (1)$$

$$\frac{d\,Q\,(t)}{d\,t} = \sim \quad (t - \text{«})^{\;-1}\,Q\,(t) - \text{¯}\,Q\,(t) - (a+b\,t) \qquad \text{«} \le t \le T \qquad \dots (2)$$

Using boundary conditions

$Q\,(0) = S,\ Q\,(\text{«}) = S_1$ and $Q\,(T) = 0$

the solutions of equation (1) and (2) are given by

$$Q\,(t) = S - (a + S\,\text{¯})\,t + \left(\frac{a\,\text{¯} - b}{2}\right)t^2 + \frac{b\,\text{¯}}{6}\,t^3 + S \sim t \;\; - \frac{a \sim \;\; t^{\;+1}}{+\,1} - \frac{b \sim \;\; t^{\;+2}}{2\,(\;\;+2)}$$

$$0 \le t \le \text{«} \qquad \dots(..3)$$

$$Q\,(t) = \begin{bmatrix} \left\{ \text{¦}\,(T - \text{«}) + \left(\frac{\text{¦}\,\text{¯} + b}{2}\right)(T-\text{«})^2 + \frac{b\,\text{¯}}{3}(T-\text{«})^3 - \frac{\sim\,\text{¦}\,(T-\text{«})^{\;+1}}{+\,1} - \frac{b \sim \;(T-\text{«})^{\;+2}}{+\,2} \right\} \\ - \left\{ \text{¦} \;+\; \text{¦}\,\text{¯}\;\;(T-\text{«}) + \frac{b\,\text{¯}}{2}(T-\text{«})^2 \right\}(t-\text{«}) + \left(\frac{\text{¦}\,\text{¯} - b}{2}\right)(t-\text{«})^2 + \frac{b\,\text{¯}}{6}(t-\text{«})^3 \\ + \left\{ \sim\;\text{¦}\,(T-\text{«}) + \frac{b \sim}{2}(T-\text{«})^2 \right\}(t-\text{«}) \;\; - \frac{\sim \;\;\text{¦}}{+\,1}(t-\text{«})^{\;+1} - \frac{b \sim}{2\,(\;\;+2)}(t-\text{«})^{\;+2} \end{bmatrix}$$

$$\text{«} \le t \le T \qquad \dots (4)$$

where $a + b\,\text{«} = \text{¦}$

Substituting $Q\,(\text{«}) = S_1$ in equation (3), we get

$$S_1 = S\,(1 - \text{¯}\;\text{«} \;+ \sim \;\text{«}\;\;) - a\,\text{«} + \left(\frac{a\,\text{¯} - b}{2}\right)\text{«}^{\,2} + \frac{b\,\text{¯}}{6}\,\text{«}^{\,3} - \frac{a \sim \;\; \text{«}^{\;+1}}{+\,1} - \frac{b \sim \;\; \text{«}^{\;+2}}{2\,(\;\;+2)} \qquad \dots (5)$$

Substituting $Q\,(\text{«}) = S_1$ in equation (4), we get

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$$S_1 = \left\{ ¦\,(T-«) + \left(\frac{¦¯+b}{2}\right)(T-«)^2 + \frac{b¯}{3}(T-«)^3 - \frac{~¦\,(T-«)^{+1}}{+1} - \frac{b~\,(T-«)^{+2}}{+2} \right\} \quad \ldots (6)$$

Eliminating S_1 from equation (5) and (6), we get

$$S = \frac{1}{\left(1-¯«+~«\right)}\left[\begin{array}{l} \left\{\begin{array}{l} ¦\,(T-«) + \left(\frac{¦¯+b}{2}\right)(T-«)^2 + \frac{b¯}{3}(T-«)^3 - \frac{~¦\,(T-«)^{+1}}{+1} \\ -\frac{b~\,(T-«)^{+2}}{+2} \end{array}\right\} \\ +a« - \left(\frac{a¯-b}{2}\right)«^2 - \frac{b¯}{6}«^3 + \frac{a~\,«^{+1}}{+1} + \frac{b~\,«^{+2}}{2(+2)} \end{array}\right] \ldots \quad (7)$$

Based on the assumptions and description of the model, the total cost TC consists of the following cost components:

I. Inventory Holding Cost (IHC)

The holding cost for carrying inventory over the period [0, T] is $IHC = \int_0^T (h+rt)\,Q(t)\,dt$

$$\Rightarrow IHC = \left\{\begin{array}{l} h\left[\begin{array}{l} S« - \frac{(a+S¯)«^2}{2} + \left(\frac{a¯-b}{2}\right)\frac{«^3}{3} + \frac{b¯}{6}\left(\frac{«^4}{4}\right) + \frac{S~\,«^{+1}}{+1} - \frac{a~\,«^{+2}}{+2} \\ -\frac{b~\,«^{+3}}{2(+2)(+3)} \end{array}\right] \\ +r\left[\begin{array}{l} \frac{S«^2}{2} - \frac{(a+S¯)«^3}{3} + \left(\frac{a¯-b}{2}\right)\frac{«^4}{4} + \frac{b¯}{6}\left(\frac{«^5}{5}\right) + \frac{S~\,«^{+2}}{+2} - \frac{a~\,«^{+3}}{+3} \\ -\frac{b~\,«^{+4}}{2(+2)(+4)} \end{array}\right] \\ +(h+r«)\left[\begin{array}{l} \left\{\begin{array}{l} ¦\,(T-«) + \left(\frac{¦¯+b}{2}\right)(T-«)^2 + \frac{b¯}{3}(T-«)^3 - \frac{~¦\,(T-«)^{+1}}{+1} \\ -\frac{b~\,(T-«)^{+2}}{+2} \end{array}\right\}(T-«) \\ -\left\{¦+¦¯(T-«)+\frac{b¯}{2}(T-«)^2\right\}\frac{(T-«)^2}{2} + \left(\frac{¦¯-b}{2}\right)\frac{(T-«)^3}{3} + \frac{b¯}{6}\frac{(T-«)^4}{4} \\ +\left\{~¦(T-«)+\frac{b~}{2}(T-«)^2\right\}\frac{(T-«)^{+1}}{+1} - \frac{~¦}{+1}\frac{(T-«)^{+2}}{+2} \\ -\frac{b~}{2(+2)}\frac{(T-«)^{+3}}{+3} \end{array}\right] \\ +r\left[\begin{array}{l} \left\{\begin{array}{l} ¦\,(T-«) + \left(\frac{¦¯+b}{2}\right)(T-«)^2 + \frac{b¯}{3}(T-«)^3 - \frac{~¦\,(T-«)^{+1}}{+1} \\ -\frac{b~\,(T-«)^{+2}}{+2} \end{array}\right\}\frac{(T-«)^2}{2} \\ -\left\{¦+¦¯(T-«)+\frac{b¯}{2}(T-«)^2\right\}\frac{(T-«)^3}{3} + \left(\frac{¦¯-b}{2}\right)\frac{(T-«)^4}{4} + \frac{b¯}{6}\frac{(T-«)^5}{5} \\ +\left\{~¦(T-«)+\frac{b~}{2}(T-«)^2\right\}\frac{(T-«)^{+2}}{+2} - \frac{~¦}{+1}\frac{(T-«)^{+3}}{+3} - \frac{b~}{2(+2)}\frac{(T-«)^{+4}}{+4} \end{array}\right] \end{array}\right\} \ldots (8)$$

II. Amelioration Cost (AMC)

The amelioration cost over the period [0, T] is $AMC = C_a\left[\int_0^« ~\,t^{-1}\,Q(t)\,dt + \int_«^T ~\,(t-«)^{-1}\,Q(t)\,dt\right]$

$$\Rightarrow AMC = C_a \sim \left[\left(\frac{S «}{\ } - \frac{a «^{+1}}{+1} - \frac{b «^{+2}}{2(\ +2)} \right) + \left(¦ (T-«) + \frac{b(T-«)^2}{2} \right) \frac{(T-«)}{\ } - \frac{¦(T-«)^{+1}}{+1} - \frac{b(T-«)^{+2}}{+2} \right]$$

… (9)

III. Deterioration Cost (DC)

The deterioration cost during the period [0,T] is

$$DC = C_d \, ¯ \left[\int_0^{«} Q(t)\,dt + \int_{«}^{T} Q(t)\,dt \right]$$

$$\Rightarrow DC = C_d \, ¯ \left[\left(S « - \frac{a «^2}{2} - \frac{b «^3}{6} \right) + \left(¦ (T-«) + \frac{b(T-«)^2}{2} \right)(T-«) - \frac{¦(T-«)^2}{2} - \frac{b(T-«)^3}{6} \right]$$

… (10)

IV. Operating Cost (OC)

The operating cost over the period [0, T] is

OC = A … (11)

V. Production Cost (PC)

The production cost per cycle is

$$PC = C_p S$$

… (12)

Total Cost (TC)

Taking the relevant costs mentioned above, the total average cost per unit time of the system is given by

$$TC = \frac{1}{T}[IHC + AMC + DC + OC + PC]$$

$$\Rightarrow TC = \frac{1}{T} \left\{ \begin{array}{l} A + C_a \sim \left[\left(\frac{S «}{\ } - \frac{a «^{+1}}{+1} - \frac{b «^{+2}}{2(\ +2)} \right) + \left(¦ (T-«) + \frac{b(T-«)^2}{2} \right) \frac{(T-«)}{\ } - \frac{¦(T-«)^{+1}}{+1} - \frac{b(T-«)^{+2}}{+2} \right] \\ + C_d \, ¯ \left[\left(S « - \frac{a «^2}{2} - \frac{b «^3}{6} \right) + \left(¦ (T-«) + \frac{b(T-«)^2}{2} \right)(T-«) - \frac{¦(T-«)^2}{2} - \frac{b(T-«)^3}{6} \right] \\ + \left\{ \begin{array}{l} h \left[S « - \frac{(a+S¯)«^2}{2} + \left(\frac{a¯ - b}{2} \right) \frac{«^3}{3} + \frac{b¯}{6} \left(\frac{«^4}{4} \right) + \frac{S \sim «^{+1}}{+1} - \frac{a \sim «^{+2}}{+2} - \frac{b \sim «^{+3}}{2(\ +2)(\ +3)} \right] \\ + r \left[\frac{S «^2}{2} - \frac{(a+S¯)«^3}{3} + \left(\frac{a¯ - b}{2} \right) \frac{«^4}{4} + \frac{b¯}{6} \left(\frac{«^5}{5} \right) + \frac{S \sim «^{+2}}{+2} - \frac{a \sim «^{+3}}{+3} - \frac{b \sim «^{+4}}{2(\ +2)(\ +4)} \right] \\ + (h + r«) \left[\begin{array}{l} \left\{ \begin{array}{l} ¦(T-«) + \left(\frac{¦¯ + b}{2} \right)(T-«)^2 + \frac{b¯}{3}(T-«)^3 \\ - \frac{\sim ¦ (T-«)^{+1}}{+1} - \frac{b \sim (T-«)^{+2}}{+2} \end{array} \right\} (T-«) \\ - \left\{ ¦ + ¦¯(T-«) + \frac{b¯}{2}(T-«)^2 \right\} \frac{(T-«)^2}{2} + \left(\frac{¦¯ - b}{2} \right) \frac{(T-«)^3}{3} \\ + \frac{b¯}{6} \frac{(T-«)^4}{4} + \left\{ \sim ¦ (T-«) + \frac{b \sim}{2}(T-«)^2 \right\} \frac{(T-«)^{+1}}{+1} - \frac{\sim ¦}{+1} \frac{(T-«)^{+2}}{+2} \\ - \frac{b \sim}{2(\ +2)} \frac{(T-«)^{+3}}{+3} \end{array} \right] \\ + r \left[\begin{array}{l} \left\{ \begin{array}{l} ¦(T-«) + \left(\frac{¦¯ + b}{2} \right)(T-«)^2 + \frac{b¯}{3}(T-«)^3 - \frac{\sim ¦ (T-«)^{+1}}{+1} \\ - \frac{b \sim (T-«)^{+2}}{+2} \end{array} \right\} \frac{(T-«)^2}{2} \\ - \left\{ ¦ + ¦¯(T-«) + \frac{b¯}{2}(T-«)^2 \right\} \frac{(T-«)^3}{3} + \left(\frac{¦¯ - b}{2} \right) \frac{(T-«)^4}{4} + \frac{b¯}{6} \frac{(T-«)^5}{5} \\ + \left\{ \sim ¦ (T-«) + \frac{b \sim}{2}(T-«)^2 \right\} \frac{(T-«)^{+2}}{+2} - \frac{\sim ¦}{+1} \frac{(T-«)^{+3}}{+3} - \frac{b \sim}{2(\ +2)} \frac{(T-«)^{+4}}{+4} \end{array} \right] \end{array} \right. \\ + C_p S \end{array} \right\}$$

… (13)

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Our objective is to determine the optimum values μ^* and T^* of μ and T respectively so that TC is minimum. Note that values μ^* and T^* can be obtained by solving the equations

$$\frac{\partial (TC)}{\partial «} = 0 \quad \& \quad \frac{\partial (TC)}{\partial T} = 0 \qquad \text{... (14)}$$

such that

$$\left.\left|\begin{matrix} \dfrac{\partial^2 TC}{\partial «^2} & \dfrac{\partial^2 TC}{\partial « \, \partial T} \\ \dfrac{\partial^2 TC}{\partial T \, \partial «} & \dfrac{\partial^2 TC}{\partial T^2} \end{matrix}\right|\right|_{« = «^*, T = T^*} > 0 \quad \& \quad \left.\frac{\partial^2 TC}{\partial «^2}\right|_{« = «^*, T = T^*} > 0 \qquad \text{... (15)}$$

The optimal solution of the equations in (14) can be obtained by using appropriate software.

4.2 Model – II : Inventory Model With Three Time Intervals

Cycle starts with inventory level of S units. The total time is divided into three time intervals. In the first time interval [0, μ] the inventory level is decreased due to the combined effect of two parameter Weibull amelioration, deterioration and demand and reaches to S_1 at t = μ. In the second time interval [μ, t_1] the inventory level goes down and reaches to S_2 at t = t_1 due to three parameter Weibull amelioration, deterioration and demand. Demand is considered as a linear function of time in time interval [0, t_1] In the third time interval [t_1, T] inventory level becomes zero at time t = T under the effect of three parameter amelioration, deterioration and demand. Here in this time interval demand is assumed as quadratic function depending on time. Deterioration rate is constant throughout time interval [0, T]. The above mentioned inventory system is presented graphically in Fig. 2.

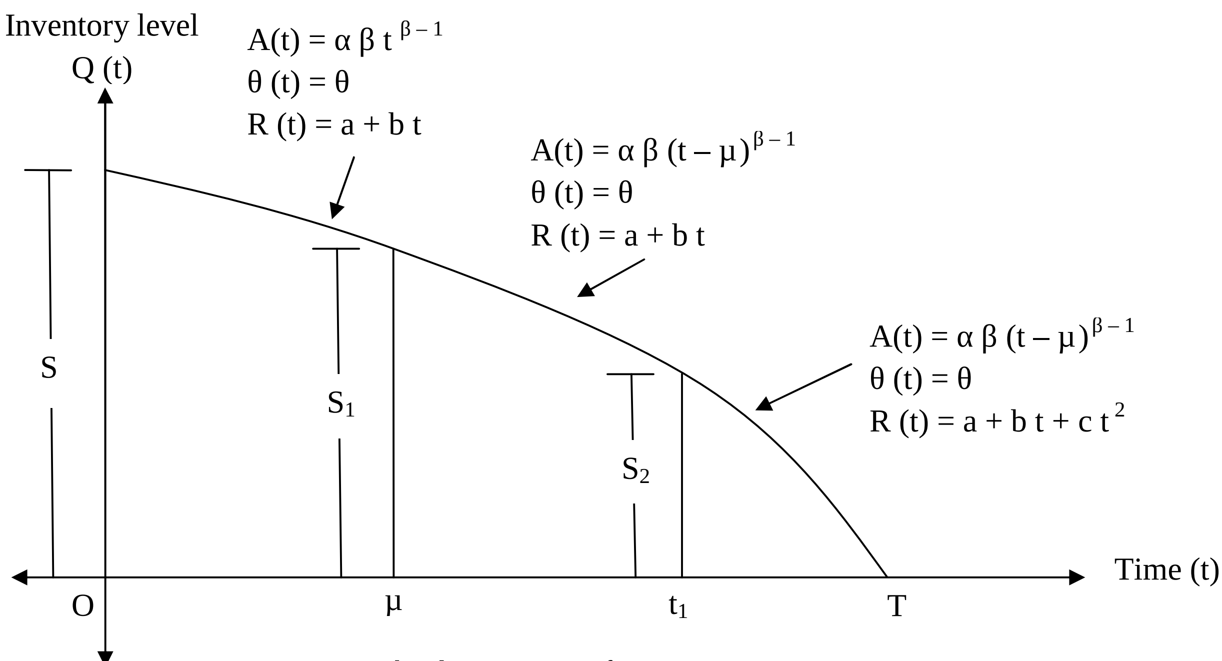

Fig. 2. Graphical presentation of Inventory System

Differential Equations pertaining to the situations as explained above are given by

$$\frac{dQ(t)}{dt} = \sim \; t^{\,-1} Q(t) - {}^{-} Q(t) - (a + b\,t) \qquad 0 \le t \le « \qquad \text{… (16)}$$

$$\frac{dQ(t)}{dt} = \sim \; (t - «)^{-1} Q(t) - {}^{-} Q(t) - (a + b\,t) \qquad « \le t \le t_1 \qquad \text{… (17)}$$

$$\frac{dQ(t)}{dt} = \sim \; (t - «)^{-1} Q(t) - {}^{-} Q(t) - \left(a + b\,t + c\,t^2\right) \qquad t_1 \le t \le T \qquad \text{... (18)}$$

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Using boundary conditions

$$Q(0) = S,\ Q(\text{«}) = S_1,\ Q(t_1) = S_2 \text{ and } Q(T) = 0$$

Solution of equation (16), (17) and (18) are given by

$$Q(t) = S - (a + S^{-})\,t + \left(\frac{a^{-} - b}{2}\right) t^2 + \frac{b^{-}}{6} t^3 + S \sim t - \frac{a \sim \quad t^{\ +1}}{+\ 1} - \frac{b \sim \quad t^{\ +2}}{2\,(\quad +\ 2)}$$

$$0 \le t \le \text{«} \qquad \dots (19)$$

$$Q(t) = \begin{cases} S_1 - (\text{¦} + S_1{}^{-})\,(t - \text{«}) + \left(\frac{\text{¦}^{-} - b}{2}\right)(t - \text{«})^2 + \frac{b^{-}}{6}(t - \text{«})^3 + S_1 \sim (t - \text{«}) \\ - \frac{\sim \quad \text{¦}\,(t - \text{«})^{\ +1}}{+\ 1} - \frac{b \sim \quad (t - \text{«})^{\ +2}}{2\,(\quad +\ 2)} \end{cases}$$

$$\text{«} \le t \le t_1 \qquad \dots (20)$$

$$Q(t) = \left[\begin{array}{l} \left\{ \begin{array}{l} x(T - \text{«}) + \left(\frac{x^{-} + y}{2}\right)(T - \text{«})^2 + \left(\frac{y^{-} + c}{3}\right)(T - \text{«})^3 + \frac{c^{-}}{4}(T - \text{«})^4 \\ - \frac{\sim\ x(T - \text{«})^{\ +1}}{+\ 1} - \frac{y \sim\ (T - \text{«})^{\ +2}}{+\ 2} - \frac{c \sim\ (T - \text{«})^{\ +3}}{+\ 3} \end{array} \right\} \\ - \left\{ x + x^{-}\ (T - \text{«}) + \frac{y^{-}}{2}(T - \text{«})^2 + \frac{c^{-}}{3}(T - \text{«})^3 \right\}(t - \text{«}) + \left(\frac{x^{-} - y}{2}\right)(t - \text{«})^2 \\ + \left(\frac{y^{-} - 2c}{6}\right)(t - \text{«})^3 + \frac{c^{-}}{12}(t - \text{«})^4 + \left\{ \begin{array}{l} \sim\ x(T - \text{«}) + \frac{\sim\ y}{2}(T - \text{«})^2 \\ + \frac{c \sim}{3}(T - \text{«})^3 \end{array} \right\}(t - \text{«}) \\ - \frac{\sim \quad x}{+\ 1}(t - \text{«})^{\ +1} - \frac{\sim \quad y}{2\,(\quad +\ 2)}(t - \text{«})^{\ +2} - \frac{c \sim}{3\,(\quad +\ 3)}(t - \text{«})^{\ +3} \end{array} \right]$$

$$t_1 \le t \le T \qquad \dots (21)$$

where $x = a + b\,\text{«} + c\,\text{«}^2$ and $y = b + 2\,c\,\text{«}$

Substituting $Q(\text{«}) = S_1$ in equation (19), we get

$$S_1 = S - (a + S^{-})\,\text{«} + \left(\frac{a^{-} - b}{2}\right)\text{«}^2 + \frac{b^{-}}{6}\text{«}^3 + S \sim \text{«} - \frac{a \sim \quad \text{«}^{\ +1}}{+\ 1} - \frac{b \sim \quad \text{«}^{\ +2}}{2\,(\quad +\ 2)}$$

$$\dots (22)$$

Substituting $Q(t_1) = S_2$ in equation (20), we get

$$S_2 = \begin{cases} S_1 - (\text{¦} + S_1{}^{-})\,(t_1 - \text{«}) + \left(\frac{\text{¦}^{-} - b}{2}\right)(t_1 - \text{«})^2 + \frac{b^{-}}{6}(t_1 - \text{«})^3 \\ + S_1 \sim (t_1 - \text{«}) - \frac{\sim \quad \text{¦}\,(t_1 - \text{«})^{\ +1}}{+\ 1} - \frac{b \sim \quad (t_1 - \text{«})^{\ +2}}{2\,(\quad +\ 2)} \end{cases} \qquad \dots (23)$$

Substituting $Q(t_1) = S_2$ in equation (21), we get

$$S_2 = \left[\begin{array}{l} \left\{\begin{array}{l} x(T-«)+\left(\frac{x^{-}+y}{2}\right)(T-«)^2+\left(\frac{y^{-}+c}{3}\right)(T-«)^3+\frac{c^{-}}{3}(T-«)^4-\frac{\sim x(T-«)^{+1}}{+1} \\ -\frac{\sim y(T-«)^{+2}}{+2}-\frac{c\sim(T-«)^{+3}}{+3} \end{array}\right\} \\ -\left\{x+x^{-}(T-«)+\frac{y^{-}}{2}(T-«)^2+\frac{c^{-}}{3}(T-«)^3\right\}(t_1-«)+\left(\frac{x^{-}-y}{2}\right)(t_1-«)^2 \\ +\left(\frac{y^{-}-2c}{6}\right)(t_1-«)^3+\frac{c^{-}}{12}(t_1-«)^4+\left\{\begin{array}{l}\sim x(T-«)+\frac{\sim y}{2}(T-«)^2 \\ +\frac{c\sim}{3}(T-«)^3\end{array}\right\}(t_1-«) \\ -\frac{\sim x}{+1}(t_1-«)^{+1}-\frac{\sim y}{2(+2)}(t_1-«)^{+2}-\frac{c\sim}{3(+3)}(t_1-«)^{+3} \end{array}\right] \quad \ldots (24)$$

Eliminating S_2 from equation (23) and (24), we get

$$S = \frac{1}{\left(\begin{array}{l}1-^{-}(t_1-«) \\ +\sim(t_1-«)\end{array}\right)}\left[\begin{array}{l} \left\{\begin{array}{l} x(T-«)+\left(\frac{x^{-}+y}{2}\right)(T-«)^2+\left(\frac{y^{-}+c}{3}\right)(T-«)^3+\frac{c^{-}}{4}(T-«)^4 \\ -\frac{\sim x(T-«)^{+1}}{+1}-\frac{\sim y(T-«)^{+2}}{+2}-\frac{c\sim(T-«)^{+3}}{+3} \end{array}\right\} \\ -\left\{x+x^{-}(T-«)+\frac{y^{-}}{2}(T-«)^2+\frac{c^{-}}{3}(T-«)^3-¦\right\}(t_1-«) \\ +\left(\frac{x^{-}-y}{2}-\frac{¦^{-}-b}{2}\right)(t_1-«)^2+\left(\frac{y^{-}-2c}{6}-\frac{b^{-}}{6}\right)(t_1-«)^3 \\ +\frac{c^{-}}{12}(t_1-«)^4+\left\{\sim x(T-«)+\frac{\sim y}{2}(T-«)^2+\frac{c\sim}{3}(T-«)^3\right\}(t_1-«) \\ -\frac{\sim(x-¦)}{+1}(t_1-«)^{+1}-\frac{\sim(y-b)}{2(+2)}(t_1-«)^{+2} \\ -\frac{c\sim}{3(+3)}(t_1-«)^{+3} \end{array}\right] \quad \ldots (25)$$

From equation (22)

$$S = \frac{1}{(1+\sim « - ^{-} «)}\left\{S_1+a« - \left(\frac{a^{-}-b}{2}\right)«^2-\frac{b^{-}}{6}«^3+\frac{a\sim «^{+1}}{+1}+\frac{b\sim «^{+2}}{2(+2)}\right\} \quad \ldots (26)$$

VI. Inventory Holding Cost (IHC)

The holding cost for carrying inventory over the period [0, T] is

$$IHC = \int_0^T (h+rt)\,Q(t)\,dt$$

$$
\Rightarrow IHC=\left\{\begin{array}{l}
h\left[S«-\frac{(a+S^{-})«^{2}}{2}+\left(\frac{a^{-}-b}{2}\right)\frac{«^{3}}{3}+\frac{b^{-}}{6}\left(\frac{«^{4}}{4}\right)+\frac{S\sim «^{\ +1}}{\ +1}-\frac{a\sim\ «^{\ +2}}{(\ +1)(\ +2)}-\frac{b\sim\ «^{\ +3}}{2(\ +2)(\ +3)}\right]\\
+r\left[\frac{S«^{2}}{2}-\frac{(a+S^{-})«^{3}}{3}+\left(\frac{a^{-}-b}{2}\right)\frac{«^{4}}{4}+\frac{b^{-}}{6}\left(\frac{«^{5}}{5}\right)+\frac{S\sim «^{\ +2}}{\ +2}-\frac{a\sim\ «^{\ +3}}{(\ +1)(\ +3)}-\frac{b\sim\ «^{\ +4}}{2(\ +2)(\ +4)}\right]\\
+(h+r«)\left[\begin{array}{l}S_{1}(t_{1}-«)-\frac{(¦+S_{1}^{-})(t_{1}-«)^{2}}{2}+\left(\frac{¦^{-}-b}{2}\right)\frac{(t_{1}-«)^{3}}{3}+\frac{b^{-}}{6}\frac{(t_{1}-«)^{4}}{4}\\
+\frac{S_{1}\sim(t_{1}-«)^{\ +1}}{\ +1}-\frac{\sim\ ¦(t_{1}-«)^{\ +2}}{(\ +1)(\ +2)}-\frac{b\sim\ (t_{1}-«)^{\ +3}}{2(\ +2)(\ +3)}\end{array}\right]\\
+r\left[\begin{array}{l}\frac{S_{1}(t_{1}-«)^{2}}{2}-\frac{(¦+S_{1}^{-})(t_{1}-«)^{3}}{3}+\left(\frac{¦^{-}-b}{2}\right)\frac{(t_{1}-«)^{4}}{4}+\frac{b^{-}}{6}\frac{(t_{1}-«)^{5}}{5}+\frac{S_{1}\sim(t_{1}-«)^{\ +2}}{\ +2}\\
-\frac{\sim\ ¦(t_{1}-«)^{\ +3}}{(\ +1)(\ +3)}-\frac{b\sim\ (t_{1}-«)^{\ +4}}{2(\ +2)(\ +4)}\end{array}\right]\\
+(h+r«)\left[\begin{array}{l}\left\{\begin{array}{l}x(T-«)+\left(\frac{x^{-}+y}{2}\right)(T-«)^{2}+\left(\frac{y^{-}+c}{3}\right)(T-«)^{3}+\frac{c^{-}}{4}(T-«)^{4}\\
-\frac{\sim x(T-«)^{\ +1}}{\ +1}-\frac{y\sim(T-«)^{\ +2}}{\ +2}-\frac{c\sim(T-«)^{\ +3}}{\ +3}\end{array}\right\}(T-t_{1})\\
-\left\{x+x^{-}(T-«)+\frac{y^{-}}{2}(T-«)^{2}+\frac{c^{-}}{3}(T-«)^{3}\right\}¶_{2}(«)+\left(\frac{x^{-}-y}{2}\right)¶_{3}(«)\\
+\left(\frac{y^{-}-2c}{6}\right)¶_{4}(«)+\frac{c^{-}}{12}¶_{5}(«)+\left\{\sim x(T-«)+\frac{\sim y}{2}(T-«)^{2}+\frac{c\sim}{3}(T-«)^{3}\right\}¶_{\ +1}(«)\\
-\frac{\sim\ x}{\ +1}¶_{\ +2}(«)-\frac{\sim\ y}{2(\ +2)}¶_{\ +3}(«)-\frac{c\sim}{3(\ +3)}¶_{\ +4}(«)\end{array}\right]\\
+r\left[\begin{array}{l}\left\{\begin{array}{l}x(T-«)+\left(\frac{x^{-}+y}{2}\right)(T-«)^{2}+\left(\frac{y^{-}+c}{3}\right)(T-«)^{3}+\frac{c^{-}}{4}(T-«)^{4}\\
-\frac{\sim x(T-«)^{\ +1}}{\ +1}-\frac{y\sim(T-«)^{\ +2}}{\ +2}-\frac{c\sim(T-«)^{\ +3}}{\ +3}\end{array}\right\}¶_{2}(«)\\
-\left\{x+x^{-}(T-«)+\frac{y^{-}}{2}(T-«)^{2}+\frac{c^{-}}{3}(T-«)^{3}\right\}¶_{3}(«)+\left(\frac{x^{-}-y}{2}\right)¶_{4}(«)+\left(\frac{y^{-}-2c}{6}\right)¶_{5}(«)\\
+\frac{c^{-}}{12}¶_{6}(«)+\left\{\sim x(T-«)+\frac{\sim y}{2}(T-«)^{2}+\frac{c\sim}{3}(T-«)^{3}\right\}¶_{\ +2}(«)\\
-\frac{\sim\ x}{\ +1}¶_{\ +3}(«)-\frac{\sim\ y}{2(\ +2)}¶_{\ +4}(«)-\frac{c\sim}{3(\ +3)}¶_{\ +5}(«)\end{array}\right]
\end{array}\right\}
$$

... (27)

VII. Amelioration Cost (AMC)

The amelioration cost over the period [0, T]

is $AMC = C_a \left[\int_0^{«} \sim \; t^{\;-1} Q(t)\, dt + \int_{«}^{t_1} \sim \; (t-«)^{\;-1} Q(t)\, dt + \int_{t_1}^{T} \sim \; (t-«)^{\;-1} Q(t)\, dt \right]$

$$\Rightarrow AMC = C_a \sim \left[\begin{array}{l} \left(\frac{S\,«}{} - \frac{a\,«^{\;+1}}{\;+1} - \frac{b\,«^{\;+2}}{2(\;+2)} \right) + \left\{ \begin{array}{l} \frac{S_1 (t_1 - «)}{} - \frac{¦\,(t_1 - «)^{\;+1}}{\;+1} \\ - \frac{b\,(t_1 - «)^{\;+2}}{2(\;+2)} \end{array} \right\} \\ + \left\{ x(T-«) + \frac{b}{2}(T-«)^2 + \frac{c}{3}(T-«)^3 \right\} ¶\;(«) - x\,¶_{\;+1}(«) \\ - \frac{y}{2}\,¶_{\;+2}(«) - \frac{c}{3}¶_{\;+3}(«) \end{array} \right] \quad \text{... (28)}$$

where $¶_k(«) = \left\{ \frac{(T-«)^k - (t_1 - «)^k}{k} \right\}$

VIII. Deterioration Cost (DC)

The deterioration cost over the period [0, T] is

$DC = C_d \,{}^{-} \left[\int_0^{«} Q(t)\, dt + \int_{«}^{t_1} Q(t)\, dt + \int_{t_1}^{T} Q(t)\, dt \right]$

$$\Rightarrow DC = C_d\,{}^{-} \left[\begin{array}{l} \left(S« - \frac{a«^2}{2} - \frac{b«^3}{6} \right) + \left\{ S_1(t_1 - «) - \frac{¦\,(t_1-«)^2}{2} - \frac{b\,(t_1-«)^3}{6} \right\} \\ + \left\{ x(T-«) + \frac{b}{2}(T-«)^2 + \frac{c}{3}(T-«)^3 \right\}(T-t_1) - x\,¶_2(«) - \frac{y}{2}\,¶_3(«) - \frac{c}{3}\,¶_4(«) \end{array} \right] \quad \text{... (29)}$$

IX. Operating Cost (OC)

The operating cost over the period [0, T] is

OC = A ... (30)

X. Production Cost (PC)

The production cost per cycle is

$PC = C_p\, S$ ... (31)

Total Cost (TC)

Taking the relevant costs mentioned above, the total average cost per unit time of the system is given by

$$TC = \frac{1}{T}\left[IHC + AMC + DC + OC + PC \right]$$

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$$
\Rightarrow TC=\frac{1}{T}\left\{\begin{array}{l}
A+C_a\sim\left[\begin{array}{l}
\left(S\text{«}-\frac{a\text{«}^{\ +1}}{+1}-\frac{b\text{«}^{\ +2}}{2(\ \ +2)}\right)+\left\{S_1(t_1-\text{«})-\frac{\text{¦}\ (t_1-\text{«})^{\ +1}}{+1}-\frac{b\,(t_1-\text{«})^{\ +2}}{2(\ \ +2)}\right\}\\
+\left\{x(T-\text{«})+\frac{b}{2}(T-\text{«})^2+\frac{c}{3}(T-\text{«})^3\right\}\P\ (\text{«})-x\P_{\ +1}(\text{«})-\frac{y}{2}\P_{\ +2}(\text{«})-\frac{c}{3}\P_{\ +3}(\text{«})
\end{array}\right]\\
+C_d-\left[\begin{array}{l}
\left(S\text{«}-\frac{a\text{«}^2}{2}-\frac{b\text{«}^3}{6}\right)+\left\{S_1(t_1-\text{«})-\frac{\text{¦}\ (t_1-\text{«})^2}{2}-\frac{b\,(t_1-\text{«})^3}{6}\right\}\\
+\left\{x(T-\text{«})+\frac{b}{2}(T-\text{«})^2+\frac{c}{3}(T-\text{«})^3\right\}(T-t_1)-x\P_2(\text{«})-\frac{y}{2}\P_3(\text{«})-\frac{c}{3}\P_4(\text{«})
\end{array}\right]\\
+\left\{\begin{array}{l}
h\left[S\text{«}-\frac{(a+S^{-})\text{«}^2}{2}+\left(\frac{a^{-}-b}{2}\right)\frac{\text{«}^3}{3}+\frac{b^{-}}{6}\left(\frac{\text{«}^4}{4}\right)+\frac{S\sim\text{«}^{\ +1}}{+1}-\frac{a\sim\ \text{«}^{\ +2}}{(\ +1)(\ +2)}-\frac{b\sim\ \text{«}^{\ +3}}{2(\ +2)(\ +3)}\right]\\
+r\left[\frac{S\text{«}^2}{2}-\frac{(a+S^{-})\text{«}^3}{3}+\left(\frac{a^{-}-b}{2}\right)\frac{\text{«}^4}{4}+\frac{b^{-}}{6}\left(\frac{\text{«}^5}{5}\right)+\frac{S\sim\text{«}^{\ +2}}{+2}-\frac{a\sim\ \text{«}^{\ +3}}{(\ +1)(\ +3)}-\frac{b\sim\ \text{«}^{\ +4}}{2(\ +2)(\ +4)}\right]\\
+(h+r\text{«})\left[\begin{array}{l}
S_1(t_1-\text{«})-\frac{(\text{¦}+S_1^{-})(t_1-\text{«})^2}{2}+\left(\frac{\text{¦}^{-}-b}{2}\right)\frac{(t_1-\text{«})^3}{3}+\frac{b^{-}}{6}\frac{(t_1-\text{«})^4}{4}\\
+\frac{S_1\sim(t_1-\text{«})^{\ +1}}{+1}-\frac{\sim\ \ \text{¦}\,(t_1-\text{«})^{\ +2}}{(\ +1)(\ +2)}-\frac{b\sim\ (t_1-\text{«})^{\ +3}}{2(\ +2)(\ +3)}
\end{array}\right]\\
+r\left[\begin{array}{l}
\frac{S_1(t_1-\text{«})^2}{2}-\frac{(\text{¦}+S_1^{-})(t_1-\text{«})^3}{3}+\left(\frac{\text{¦}^{-}-b}{2}\right)\frac{(t_1-\text{«})^4}{4}+\frac{b^{-}}{6}\frac{(t_1-\text{«})^5}{5}\\
+\frac{S_1\sim(t_1-\text{«})^{\ +2}}{+2}-\frac{\sim\ \ \text{¦}\,(t_1-\text{«})^{\ +3}}{(\ +1)(\ +3)}-\frac{b\sim\ (t_1-\text{«})^{\ +4}}{2(\ +2)(\ +4)}
\end{array}\right]
\end{array}\right.\\
+\left\{\begin{array}{l}
+(h+r\text{«})\left[\begin{array}{l}
\left\{\begin{array}{l}x(T-\text{«})+\left(\frac{x^{-}+y}{2}\right)(T-\text{«})^2+\left(\frac{y^{-}+c}{3}\right)(T-\text{«})^3+\frac{c^{-}}{4}(T-\text{«})^4-\frac{\sim x\,(T-\text{«})^{\ +1}}{+1}\\-\frac{y\sim(T-\text{«})^{\ +2}}{+2}-\frac{c\sim(T-\text{«})^{\ +3}}{+3}\end{array}\right\}(T-t_1)\\
-\left\{x+x^{-}(T-\text{«})+\frac{y^{-}}{2}(T-\text{«})^2+\frac{c^{-}}{3}(T-\text{«})^3\right\}\P_2(\text{«})+\left(\frac{x^{-}-y}{2}\right)\P_3(\text{«})+\left(\frac{y^{-}-2c}{6}\right)\P_4(\text{«})\\
+\frac{c^{-}}{12}\P_5(\text{«})+\left\{\sim x\,(T-\text{«})+\frac{\sim y}{2}(T-\text{«})^2+\frac{c\sim}{3}(T-\text{«})^3\right\}\P_{\ +1}(\text{«})-\frac{\sim\ \ x}{+1}\P_{\ +2}(\text{«})\\
-\frac{\sim\ \ y}{2(\ +2)}\P_{\ +3}(\text{«})-\frac{c\sim}{3(\ +3)}\P_{\ +4}(\text{«})
\end{array}\right]\\
+r\left[\begin{array}{l}
\left\{\begin{array}{l}x(T-\text{«})+\left(\frac{x^{-}+y}{2}\right)(T-\text{«})^2+\left(\frac{y^{-}+c}{3}\right)(T-\text{«})^3+\frac{c^{-}}{4}(T-\text{«})^4-\frac{\sim x\,(T-\text{«})^{\ +1}}{+1}\\-\frac{y\sim(T-\text{«})^{\ +2}}{+2}-\frac{c\sim(T-\text{«})^{\ +3}}{+3}\end{array}\right\}\P_2(\text{«})\\
-\left\{x+x^{-}(T-\text{«})+\frac{y^{-}}{2}(T-\text{«})^2+\frac{c^{-}}{3}(T-\text{«})^3\right\}\P_3(\text{«})+\left(\frac{x^{-}-y}{2}\right)\P_4(\text{«})+\left(\frac{y^{-}-2c}{6}\right)\P_5(\text{«})\\
+\frac{c^{-}}{12}\P_6(\text{«})+\left\{\sim x\,(T-\text{«})+\frac{\sim y}{2}(T-\text{«})^2+\frac{c\sim}{3}(T-\text{«})^3\right\}\P_{\ +2}(\text{«})-\frac{\sim\ \ x}{+1}\P_{\ +3}(\text{«})\\
-\frac{\sim\ \ y}{2(\ +2)}\P_{\ +4}(\text{«})-\frac{c\sim}{3(\ +3)}\P_{\ +5}(\text{«})
\end{array}\right]
\end{array}\right\}\\
+C_p\,S
\end{array}\right\}
$$

... (32)

Our objective is to determine optimum values μ^*, t_1^* and T^* of μ, t_1 and T respectively so that TC is minimum. Note that values μ^*, t_1^* and T^* can be obtained by solving the equations

$$\frac{\partial(TC)}{\partial \mu} = 0, \quad \frac{\partial(TC)}{\partial t_1} = 0 \quad \& \quad \frac{\partial(TC)}{\partial T} = 0 \qquad \text{... (33)}$$

such that

$$\left.\begin{array}{l} \left|\begin{array}{ccc} \dfrac{\partial^2 TC}{\partial \mu^2} & \dfrac{\partial^2 TC}{\partial \mu\, \partial t_1} & \dfrac{\partial^2 TC}{\partial \mu\, \partial T} \\ \dfrac{\partial^2 TC}{\partial t_1\, \partial \mu} & \dfrac{\partial^2 TC}{\partial t_1^2} & \dfrac{\partial^2 TC}{\partial t_1\, \partial T} \\ \dfrac{\partial^2 TC}{\partial T\, \partial \mu} & \dfrac{\partial^2 TC}{\partial T\, \partial t_1} & \dfrac{\partial^2 TC}{\partial T^2} \end{array}\right|_{\mu = \mu^*,\, t_1 = t_1^*,\, T = T^*} > 0, \quad \left|\begin{array}{cc} \dfrac{\partial^2 TC}{\partial \mu^2} & \dfrac{\partial^2 TC}{\partial \mu\, \partial t_1} \\ \dfrac{\partial^2 TC}{\partial t_1\, \partial \mu} & \dfrac{\partial^2 TC}{\partial t_1^2} \end{array}\right|_{\mu = \mu^*,\, t_1 = t_1^*,\, T = T^*} > 0 \\ \& \left.\dfrac{\partial^2 TC}{\partial \mu^2}\right|_{\mu = \mu^*,\, t_1 = t_1^*,\, T = T^*} > 0 \end{array}\right\} \qquad \text{... (34)}$$

The optimal solution of the equations in (33) can be obtained by using appropriate software.

Here it can be observed that if we consider the value of c = 0 and time $t_1 = \mu$ in Model – II then we can obtain all the results of inventory system developed in Model – I. That means Model – I is a particular case of Model – II.

The above developed Model – I is illustrated by means of the following numerical example.

5. Numerical Example

Let us consider the following numerical example to illustrate the inventory model which is developed in Model – I. We take the values of the parameters A = 200, α = 0.0001, β = 12, r = 2, h = 5, C_a = 7, C_d = 4, C_p = 15, θ = 0.01 and a = 6, b = 3 (with appropriate units of measurement). We obtain the values μ = 2.3752894493922168 units, T = 2.883728392984433 units and total cost TC = 288.970522829728 units and order quantity S = 22.7021925679148 units by using appropriate software.

6. Sensitivity Analysis

Sensitivity analysis is very important technique to identify the effect on optimal solution of the model by changing its parameter values. In this section, we study the sensitivity of total cost TC per time unit and inventory level S per unit with respect to the changes in the values of the parameters A, α, r, h, C_a, C_d, C_p, θ, a, and b.

This analysis is performed by considering 10% and 20% increase and decrease in each one of the above parameters keeping all other remaining parameter as fixed. The results are presented in the Table below. The last two columns of the table show the % change in TC and % change in S respectively as compared to the original solution corresponding to the change in parameters values, taken one by one.

Sensitivity Analysis

Parameter	**% change**	**μ**	**T**	**TC**	**% change in TC**	**S**	**% change in S**
A	– 20	2.329213082	2.724329953	274.7319052	– 4.92735989	22.10186262	– 2.6443698977
	– 10	2.355660963	2.812873221	281.9509883	– 2.42915246	22.44569414	– 1.1298398913

	10	2.391216414	2.944690317	295.8321636	2.374512344	22.9111479	0.9204191524
	20	2.404751386	2.999083175	302.5610446	4.703082375	23.0893037	1.7051707009
α	– 20	2.407145131	2.890588001	291.4619855	0.862185751	23.12168499	1.8478057571
	– 10	2.390421144	2.887308156	290.1384385	0.404164309	22.90107374	0.8760439170
	10	2.361492811	2.880039612	287.9272218	– 0.36104065	22.5214588	– 0.7961071051
	20	2.34882683	2.876344761	286.9858014	– 0.68682488	22.3560403	– 1.5247525898
r	– 20	2.397567737	2.969871403	284.0496245	– 1.70290666	22.99455265	1.2878054738
	– 10	2.386377763	2.925814105	286.544109	– 0.83967522	22.84753697	0.6402218609
	10	2.364207658	2.843171607	291.334035	0.817907709	22.5572704	– 0.6383619966
	20	2.353029573	2.803727591	293.6387984	1.615485025	22.4114329	– 1.2807559071
h	– 20	2.390183971	2.940374343	277.073882	– 4.11690463	22.89668949	0.8567318870
	– 10	2.382765557	2.911823598	283.0370605	– 2.0533106	22.79976997	0.4298148886
	10	2.36770347	2.855900668	294.8751668	2.04333783	22.60328783	– 0.4356615983
	20	2.359945436	2.828134547	300.7516275	4.076922625	22.50225869	– 0.8806809031
Ca	– 20	2.403229276	2.99322678	285.1455482	– 1.32365563	23.07047296	1.6222238833
	– 10	2.389886451	2.939655841	287.1158035	– 0.64183685	22.89427346	0.8460896026
	10	2.359108799	2.824835249	290.7039713	0.599870337	22.49009612	– 0.9342553648
	20	2.340842407	2.762040304	292.3090179	1.155306429	22.25169013	– 1.9844005692
Cd	– 20	2.375309624	2.883935556	288.8813993	– 0.03084175	22.70291921	0.0032007655
	– 10	2.375299542	2.883831962	288.9259613	– 0.01542078	22.70255585	0.0016002252
	10	2.375279346	2.883624849	289.0150838	0.01542058	22.70182935	– 0.0015999108
	20	2.375269233	2.883521329	289.0596441	0.030840963	22.70146621	– 0.0031995077
Cp	– 20	2.338097994	2.753018295	265.0629912	– 8.27334614	22.2163179	– 2.1402103087
	– 10	2.357852549	2.82045069	277.0907124	– 4.11108037	22.47387472	– 1.0057083762
	10	2.390879081	2.943481012	300.7109915	4.062860312	22.90710007	0.9025890243
	20	2.404964898	3.000197172	312.3199711	8.080218041	23.09287373	1.7208961879
θ	– 20	2.37325714	2.881217456	288.8229283	– 0.05107598	22.67798319	– 0.1066389553
	– 10	2.374276296	2.882479034	288.8968255	– 0.0255034	22.69012277	– 0.0531657932
	10	2.376296691	2.884965739	289.0440213	0.025434591	22.7141937	0.0528633188
	20	2.377298112	2.886191271	289.117322	0.050800734	22.72612725	0.1054289302
a	– 20	2.373577275	2.873175618	263.7883916	– 8.71442906	19.83249606	– 12.6406139106
	– 10	2.374450268	2.878585808	276.3800412	– 4.35701244	21.26704195	– 6.3216388102
	10	2.37609525	2.888620952	301.5599253	4.356638984	24.13789283	6.3240599304
	20	2.376868469	2.893279822	314.1483286	8.712932216	25.57409395	12.6503260701
b	– 20	2.410054689	3.026960616	269.8510149	– 6.61642155	21.418355	– 5.6551258828
	– 10	2.393040742	2.954280247	279.5066594	– 3.27502727	22.07690751	– 2.7542937037
	10	2.35618874	2.813151301	298.2518877	3.211872536	23.2846059	2.5654496925
	20	2.334659566	2.739473701	307.3522889	6.361121491	23.80632843	4.8635648606

Table: Sensitivity analysis of parameters considered in defining inventory model

7. Graphical Presentation

Graphical presentation of the above sensitivity analysis is shown in Fig. 3.

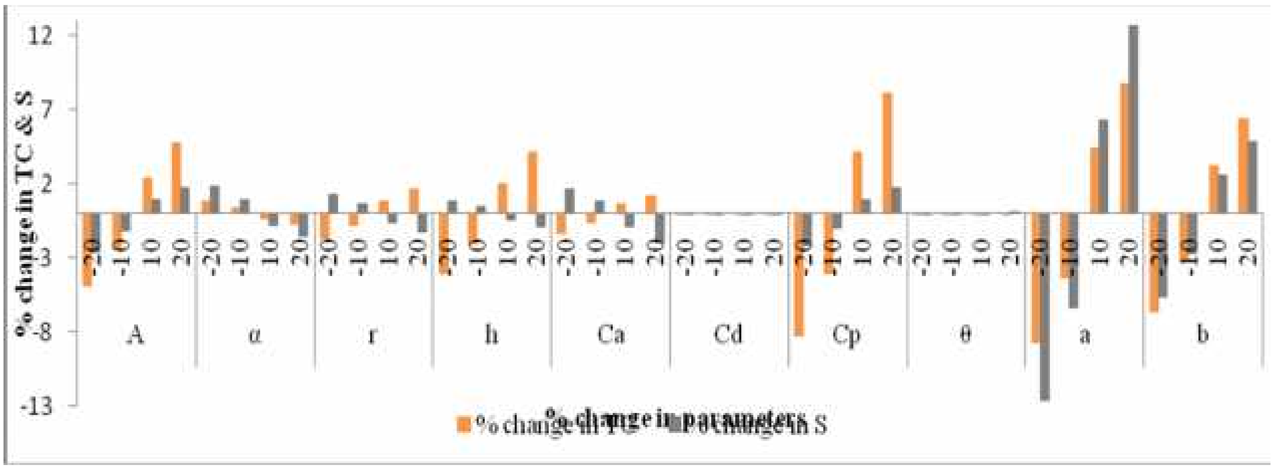

Fig. 3. Graphical presentation of the above partial sensitivity analysis

8. Conclusion

From The Above Sensitivity Analysis we may conclude that the total cost TC per time unit is highly sensitive to the changes in the values of the parameters C_p, a, b moderately sensitive to the changes in the values of the parameters A, h, C_a and less sensitive to the changes in the values of the parameters r, α, C_d, θ.

We may also conclude that the inventory level S per unit is highly sensitive to the changes in the values of the parameters a, b moderately sensitive to the changes in the values of the parameters A, h, C_a, C_p, r, α and less sensitive to the changes in the values of the parameters C_d, θ.

Moreover, it can also be observed from Fig. 3. that there is simultaneous change in total cost TC per time unit and inventory level S per unit as the values of the parameters A, α, a, b and C_p increase or decrease whereas opposite change can be found in total cost TC per time unit and inventory level S per unit as the values of the parameters r, h, C_a, C_d and θ.

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