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COUNTING VISIBLE POINTS IN MORE GENERAL REGIONS

Dr. A. Vethamanickam* **K.E.Usha Nirmala Kumari****
*Associative Professor, Rani Anna Government College for Women, Thirunelveli.
**Research Scholar, H.H.The Rajah's College, Pudukkottai.

Abstract
We say that one lattice point is visible from another if no third lattice point lies on the line joining them. A lattice point visible from the origin is called a visible point. To find the visible points is very useful in probability theory. In this paper we deal with counting visible points in more general regions.

Introduction
Returning to the study of the density of V in L, we may ask for the limiting fraction of visible points in an expanding region of more general shape than a rectangle. Under what conditions can we be sure the limit exists, and if it exists must it be equal to $\frac{6}{\pi^2}$.

One's imagination immediately pictures an amoeba – like region expending thrusting forth long tentacles toward special lattice points, and clearly nothing could be concluded if such pathology were allowed. We will discuss the case in which a region R which has a positive area expands by linear magnification about the origin. Depending on whether the origin is contained in the interior of R, or lies on its boundary, or is exterior to R, the expanding region will envelop the whole plane, or a portion of it, or will disappear into the distance.

To begin, R may be an arbitrary bounded point set. If t > o, let tR denote the image of R under the mapping f(z) = tz. Let N (tR) be the number of lattice points, excluding the origin, In tR.

Let N' (tR) be the number of visible points in tR. From the known result $\frac{1}{xy}N'(X,Y) = \frac{6}{\pi^2} + 0\left(\frac{\log z}{z}\right)$ Where z = min (x, y), which counts the visible points of a rectangle, is readily extended to give the following relationship between N and N'.

Theorem 1.1

$$N(R) = \sum_{k=1}^{\infty} N'\left(\frac{R}{k}\right)$$

$$N'(R) = \sum_{k=1}^{\infty} \mu(k) N\left(\frac{R}{k}\right).$$

Proof:
Both sums are finite since R/k eventually contains no lattice point except perhaps the origin. Theorem 1.1 is formally identical to a known inversion formula satisfied by the Mobious function.

$$N(R) = \sum_{\substack{(m,n)\varepsilon R \\ (m,n)\neq(0,0)}} 1$$

$$= \sum_{d=1}^{\infty} \sum_{\substack{(m,n)\varepsilon R \\ (m,n)=d}} 1$$

$$= \sum_{d=1}^{\infty} \sum_{\substack{(m,n)\varepsilon \frac{R}{d} \\ (m,n)=1}} 1$$

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$$= \sum_{d=1}^{\infty} N\left(\frac{R}{d}\right)$$

$$N'(R) = \sum_{\substack{(m,n)\varepsilon R \\ (m,n)=1}} 1$$

$$= \sum_{(m,n)\varepsilon R} \sum_{d/(m,n)} \mu(d)$$

$$= \sum_{d=1}^{\infty} \mu(d) \sum_{\substack{(m,n)\varepsilon R \\ (m,n)=d}} 1$$

$$= \sum_{d=1}^{\infty} \mu(d) \sum_{\substack{(m,n)\varepsilon R \\ (m,n)=d}} 1$$

$$= \sum_{d=1}^{\infty} \mu(d) N\left(\frac{R}{d}\right)$$

From this the number N' (X, Y) of visible points in Q(x, y) is exactly.

$\sum_{k=1}^{\infty} \mu(k) \left[\frac{x}{k}\right]\left[\frac{y}{k}\right]$. Here the sum is finite, since the terms are zero when k>min(x,y).is, of course, the case R = Q(X, Y).

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$\sum_{k=1}^{\infty} \mu(k) \left[\frac{x}{k}\right]\left[\frac{y}{k}\right]$. Here the sum is finite, since the terms are zero when k>min(x, y).

From the known result Everyunimodular transformation of L maps V onto itself.

We can deduce that as $t \to \infty$, N' (tR)N(tR)$\to 6/\pi^2$, under certain restrictions on R. Hereafter we shall write N(t) and N'(t) instead of N(tR) and N'(tR). Define N,(t) to be the number of lattice squares contained entirely in tR, and N_2(t) to be the number of lattice squares having at least one point in tR. Then we have the inequalities.

$$N_1(t) \leq N(t) \leq N_2(t). \qquad \to \quad \text{(I)}$$

Note

N_1 (t) = no lattice point.
N_2 (t) = contain at least one lattice point.

We will now assume that R possesses a positive area A(R) given by

$$A(R) = \lim_{t \to \infty} \frac{N_1(t)}{t^2} = \lim_{t \to \infty} \frac{N_2(t)}{t^2} \qquad \to \quad \text{(II)}$$

Then A(tR) = t^2 A(R), and we will write A(t) for A(tR) and A for A(1). From (E) and (F) we have

$$N(t) \sim At^2. \qquad \rightarrow \qquad \text{(III)}$$

Let e=l.u.b. {t/N (t) = 0}, which is finite because of (G) and greater than zero because R is bounded.

For any t > c. Let k(t) be the largest integer k such that N(t/k) > o. We note that.

$$k(t) = \left[\frac{t}{c}\right] \ or \ \left[\frac{t}{c}\right] - 1. \qquad \rightarrow \qquad \text{(IV)}$$

For. By definition we must have

$$N_1(t) \leq N(t) \leq N_2(t)$$

$$\frac{t}{k(t)+1} \leq 0 \leq \frac{t}{k(t)}$$

And hence

$$k(t) \leq \frac{t}{o} \leq k(t)+1.$$

We may choose the size of R to be such that c=1.

Finally, let P (t) = N (t) – A (t).

Then,

$$\frac{N'(t)}{N(t)} + 0(1)$$

$$\frac{N'(t)}{At^2} = \frac{1}{At^2}\sum_{k=1}^{k(t)} \mu(k) N\left(\frac{t}{k}\right)$$

$$= \frac{1}{At^2}\sum_{k=1}^{(t)} \mu(k)\left(A\left(\frac{t}{k}\right) + P\left(\frac{t}{k}\right)\right)$$

$$= \frac{1}{At^2}\sum_{k \leq t} \frac{\mu(k)A(t^2)}{k^2} + \frac{1}{At^2}\sum_{k \leq t} \mu(k) P\left(\frac{t}{k}\right).$$

$$(\because M(t) = \sum_{k=1}^{\infty} \quad \Sigma \quad \mu(k) N\left(\frac{A}{k}\right)).$$

$$= \sum_{k \leq t} \frac{\mu(k)}{k^2} + \frac{1}{At^2}\sum_{k \leq t} \mu(k) P\left(\frac{t}{k}\right).$$

Hence proved.

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Result: 1.1

If R is bounded and has a positive area, then N'(t)/N(t) $\to \frac{6}{\pi^2}$ if, and only if.

$$\sum_{k\leq t}\mu(k)P\left(\frac{t}{k}\right)=0(t^2).$$

We shall be content with a very generous sufficient condition.

Theorem 1.2

The condition P(t) = 0(t) implies

$$\sum_{k\leq t}\mu(k)P\left(\frac{t}{k}\right)=0(t^2).$$

Proof:

If R is a region whose boundary is a rectifiable curve.
If $|P(t)| \leq Mt$ for all t > 0, then]

$$\left|\sum_{k\leq t}\mu(k)P\left(\frac{t}{k}\right)\right| \leq Mt\sum_{k\leq t}\frac{1}{k}\leq M'tlog\ t=0(t^2)$$

Suppose R is bounded by a curve C of length S. Then C can pass through no more than 4[S]+4 lattice squares.

For suppose we cut C into [S] arcs of unit length plus one are of length {S}, and arrange these individually in the lattice to maximize the total number of squares passed through. Each segment can pass through at most four squares, giving a maximum total of 4[S]+4, and this is also a maximum total for the original curve C since it constituted one of the possible arrangements of the segments. Thus the boundary tC of tR can pass through at most 4[tS]+4 squares.

But since $N_1(t) \leq N(t) \leq N_2$ (t) and $N_1(t) \leq$ A(t $\leq N_2$ (t), we have $|P(t)|=|N(t) - A(t)| \leq N_2(t) - N_1(t)$, and $N_2(t) - N_1(t)$ is the number of squares with at least one point inside and at least one point outside or tR. The boundary curve tC must pass through each of these squares, so there cannot be more than 4[tS]+4 of them.

Thus $|P(t)| \leq N_2(t) - N_1(t) \leq 4[tS]+4=O(t)$.

We note that the condition P(t) = 0(t) is satisfied, in particular, if R is convex, for bounded convex sets have rectifiable boundaries.

Equations (II) and (III) do not furnish a direct generalization of result $\frac{1}{xy}N'(X,Y)=\frac{6}{\pi^2}+0\left(\frac{\log z}{z}\right)$Where z = min (x, y) because we have considered only the case in which R expands by linear magnification about the origin. In a similar manner we could treat the case of expansion under the transformation f(x, y) = (t_1 x_1 t_2 y), of which the rectangle in the mean of $F_s(X)$ is $\frac{\zeta(s-1)}{\zeta(s)}$ if s≥3 and infinite if s=2 the variance is $\frac{\zeta(s-2)}{\zeta(s)}-\frac{\zeta^2(s-1)}{\zeta^2(s)}$ is a special case. However, we shall not pursue the details here.

References

1. G.H Hardy and M. Wright. An Introduction the theory of numbers, 3rd Edition, Oxford (1954)
2. F. Mertens, Journals $f\ddot{u}r$ math, 77, 289 – 338.
3. W.J. Leveque, Topics in number theory, Volume – I Addison – Wesley.
4. J. Christopher, Am. Math. Monthly, 63. No.6, (399 – 401)
5. H.L. Alder, Am. Math, monthly, 65, No.9, (690 – 692)
6. V. A. Golubev, Mathesis, Tome LXVII, 11 – 20.
7. D.R. Anderson and T.M. Apostol, Duke math, Journal, 20, No.2, (211 – 216).